Calculus 3

Course Number:	2105
Course Level:	College Intending
Grade:	11 – 12
Time/Credit:	83 minutes daily; 18 weeks
	1 credit
Revision:	Fall 2025 – Summer 2026
Board Approval:	

Central Bucks School District
20 Welden Drive
Doylestown, PA 18901

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K – 12 Mathematics Program Learning Principles

Teaching and Learning Principles:

1. Teaching mathematics for understanding requires providing experiences that stimulate students' curiosity and build confidence in investigating, problem solving, and communicating (speaking, reading, writing, representing, and listening).
2. Differentiated instruction addresses student strengths, interests, and learning styles.
3. Instruction should be appropriately paced to make the curriculum accessible to all students.
4. High expectations with strong support from teachers, parents and administrators motivate all students to achieve excellence in mathematics.
5. To create new understanding, students must be actively engaged in learning experiences designed to connect to their prior knowledge.
6. Presenting new ideas within rich and meaningful contexts helps learners to transfer and use knowledge meaningfully.
7. Making connections both within and outside of school helps learners to transfer and use knowledge meaningfully.
8. The learning process is enhanced when new ideas are presented using manipulatives, computers, calculators and real-time data collection devices.
9. Teacher collaboration and reflection enrich learning experiences for students.
10. Assessment is an ongoing process that guides instruction and monitors student progress to include mastery of mathematics content and higher level thinking skills.

Documents Considered:

(2000). Principles and Standards for School Mathematics. Reston, VA: National Council of Teachers of Mathematics.

Kilpatrick, J., Swafford, J., & Findell, B. (2001). *Adding It Up: Helping Children Learn Mathematics.* Washington, DC: National Academy Press.

Marzano, R., & Pickering, D. (1997). *Dimensions of Learning.* Alexandria,VA: Association for Supervision and Curriculum Development; Aurora, CO: Mid-continent Regional Educational Laboratory.

Tomlinson, Carol Ann. (1999). *The Differentiated Classroom: Responding to the Needs of All Learners.* Alexandria, VA: Association for Supervision and Curriculum Development.

Zemelman, S., Daniels, H., & Hyde, A. (1998). *Best practice: New standards for teaching and learning in America's schools*. Portsmouth, NH: Heinemann.

K – 12 Mathematics Program Enduring Understandings

Students will understand that:

The Nature of Mathematics

- Mathematics is a language consisting of carefully defined terms and symbols.
- Mathematics is a tool used to make informed decisions and to solve problems in everyday life.

Numbers and Operations

- Numbers have multiple equivalent representations.
- Numbers can be classified into various number systems.
- Calculations are performed using a predetermined order of operations.

Measurement

- All measurements using a tool other than counting are estimates with an accepted level of error.
- All measurements consist of exactly three parts: number, unit, and precision.
- Changing a linear dimension of a figure affects the perimeter and circumference differently than it affects the area (two-dimensional shapes) and volume (three-dimensional shapes).

Geometry

- Points, lines, and planes are the building blocks of geometry.
- Postulates, theorems, definitions, and properties are used to justify reasoning in a direct proof and to establish relationships involving two and three-dimensional figures.
- Logical thinking and critical reasoning skills are important aspects of problem solving.
- Shapes are quantified using characteristics such as dimensionality, side measures, vertices, faces, and edges, area, perimeter and volume.
- The Pythagorean theorem and trigonometric ratios are used to find missing quantities in right triangles.
- Concepts of congruency and similarity are used to relate and compare 2 and 3- dimensional figures.
- Ratios and proportions are used to model geometric similarity.

Algebraic Concepts

- Algebraic expressions, equations, functions (linear, absolute value, quadratic, polynomial, exponential, and logarithmic), and inequalities are used to model relationships between quantities in real-world situations.
- Basic algebraic rules and properties determine how expressions are simplified and how equations are solved.
- Problems using a constant rate of change can be modeled using proportional reasoning.
- Functions are represented using: verbal models, tables, equations, and graphs.
- Systems of equations can be solved both graphically and algebraically and are used to model real-life problems.

Data Analysis and Probability

- Data is collected, organized, and displayed as a tool for answering questions about a population.
- Interpretations and predictions are influenced by the statistical measure used to collect a set of data and by the representation that is used to display the given data.
- Probability is used to make predictions and inferences.
- Probability can be applied to geometric situations.

K – 12 Mathematics Program Essential Questions

The Nature of Mathematics

- Is mathematics a language?
- When is the correct solution not the answer to a problem?

Numbers and Operations

- What is a reasonable answer?
- Why are there different number systems?
- What does it mean to be simplified?
- Is there more than one way to simplify an expression?

Measurement

- What does it mean to be accurate?
- How do you measure?
- How does changing a dimension change a figures length, total area and volume?

Geometry

- Why do all statements need justification?
- What is the logical progression of statements in a proof?
- How do you name a geometric figure?
- Why are points, lines, and planes so important in Euclidean geometry?
- Why are naming conventions important?
- How and when is the Pythagorean Theorem used?
- What do similarity and congruency statements tell you about polygons?

Algebraic Concepts

- What does solve mean?
- Is your solution always the appropriate answer to a problem?
- How do expressions and equations differ?
- What makes a relationship linear?
- How can a line be used to make predictions?
- When is it necessary to use a system of equations?
- Why would you represent a function in different ways?

Data Analysis and Probability

- What is the purpose of collecting data?
- How is data used?
- How do we make predictions based on probability?

Calculus 3 Overview

Desired Results

2105 Calculus 3
(18 weeks, 1 credit)
Calculus 3 will extend the concepts learned in single-variable calculus to functions of several variables and three-dimensional space. Topics will include vectors and the geometry of space, vector-valued functions, partial derivatives, and multiple integrals. Students will study vector calculus, including line and surface integrals, as well as the fundamental theorems of vector calculus, such as Green's Theorem, Stokes' Theorem, and the Divergence Theorem. Additional topics include the use of different coordinate systems (polar, cylindrical, and spherical), applications of multiple integrals, and second-order differential equations. This course parallels the third semester of most college calculus sequences and emphasizes real-world applications, mathematical reasoning, and the use of technology for visualization and problem-solving.

Acceptable Evidence

Final Exam: A cumulative multiple-choice exam is given at the end of the course. The final exam grade counts as 10% of the final course grade.

Calculus 3 Pacing Guide

Content	Blocks
Unit 1: Review of Single Variable Calculus	**5**
- The derivative of a function expresses the instantaneous rate of change of the function. - Rules for differentiating a function, differentiation techniques for various function types and implicit differentiation. - The integral of a function expresses the net change of a function over time. - Rules for integrating a function, integration techniques for various function types, integration by parts, tabular integration, integration by substitution. - Using conceptual ideas of calculus to model and explore real-world situations.	3
Remediation, enrichment	1
Review and Assessment	2
Unit 2: Three-Dimensional Space; Vectors (Chapter 11)	**10**
- The geometry of three-dimensional space, representing lines and planes in three-dimensional space. - Constructing the equations of three-dimensional figures in space, points, lines, and planes, as well as the equations of quadric surfaces. - Definition of a vector and the properties of vector addition and multiplication. - The dot product and cross product of two vectors can be used to create orthogonal projections, determine the angle between two vectors, or create the perpendicular to the intersection of the vectors. - The cross product is the vector perpendicular to two given vectors, whose length is the area of the parallelogram determined by the vectors.	7
Remediation, enrichment	1
Review and Assessments	2
Unit 3: Vector-Valued Functions (Chapter 12)	**12**
- Use the concept and properties for determining the derivative and integral of single variable functions to determine the derivative of vector functions. - Use computers (graphing calculators) to draw space curves given their functions. - Determine the curvature and arc length of a curve. - Use vector functions to describe the motion of an object in space.	9
Remediation, enrichment	1
Review and Assessment	2
Unit 4: Partial Derivatives (Chapter 13)	**12**
- Use level curves to visualize functions of several variables. - Extend the ideas of implicit differentiation from single variable calculus to determine particular and partial derivatives for functions of several variables. - Find the limit of a multivariable function. - Use previous rules for differentiation, including the Chain Rule, to determine partial derivatives for functions of several variables. - Use fundamental concepts of limits and continuity to determine the limit and continuity for functions of several variables.	9

- Determine tangent lines, planes, and linear approximations of functions of several variables. - Use partial derivatives to create directional derivatives and the gradient vector of a function. - Use partial derivatives to determine classifying characteristics of three-dimensional surfaces, including maximums, minimums, and saddle points. - Utilize Lagrange multipliers in addition to partial derivatives to determine classifying characteristics of three-dimensional surfaces. -	
Remediation, enrichment	1
Review and Assessment	2
Unit 5: Multiple Integrals (Chapter 14)	**15**
- Extend the ideas of integration from single variable calculus to determine multiple integrals for functions of several variables. - Use double integrals to determine the volume of a three-dimensional shape. - Double integrals can be used to determine density, mass, moments, centers of mass, and probabilistic equations such as the joint density function. - Use iterated integrals to determine the surface area of a quadric surface. - The concept of multiple integrals can be extended from Cartesian coordinates to polar, cylindrical, and spherical coordinates. - The substitution method from single-variable calculus is extrapolated to the change-of-variables method in multiple integrals to simplify integration. -	12
Remediation, enrichment	1
Review and Assessments	2
Unit 6: Topics in Vector Calculus (Chapter 15)	**20**
- Integrating a function over a curve C in place of a set interval [a, b] produces the line integral of the function. - The fundamental theorem for line integrals relates a function's gradient and the integral of the function along a smooth curve. This creates the relationship that the line integral of the gradient of a function is the net change in the function. - Green's Theorem provides the relationship between a line integral around a simple closed curve and a double integral over the plane bounded by the curve. - The curl of a function is the vector field defined by the cross product of the function and its gradient. - The divergence of a function is the scalar field defined by the dot product of the function and its gradient.	15
Remediation, enrichment	1
Review and Assessments	4
Unit 7: Second Order Differential Equations	**12**
- A review of first-order differential equations, including first-order linear differential equations. - A second order linear differential equation is of the form $P(x)\left(\frac{d^2y}{dx^2}\right) + Q(x)\left(\frac{dy}{dx}\right) + R(x)y = G(x)$ where P, Q, R, G are continuous functions. - Second-order differential equations are either homogeneous, in the case , or nonhomogeneous. - The method of undetermined coefficients is used to solve nonhomogeneous linear equations.	8

- Second-order differential equations are used to model various real-world situations, such as vibrations of strings and electric circuits. - The solution to a second order differential equation can be represented as an infinite series.	
Remediation, enrichment	1
Review and Assessments	3
Final Exam Review	**2**
Final Exam	**1**

Required and Supplementary Texts/Software

Required Text:

Calculus: Early Transcendentals, 12th Edition (2022), Howard Anton; Irl C. Bivens; Stephen Davis

Supplementary Texts/Software:

Desmos

Graphing Calculator – The graphing calculator is an essential part of this course. All students need to have access to a graphing calculator for this course.

Calculator Recommendation

The secondary mathematics department of Central Bucks School District recognizes the use of calculators as a valuable tool for learning in the mathematics classroom. Graphing calculators will be used daily during this course. In certain advanced courses, graphing calculators with specific capabilities are important for daily classroom performance and are required for advanced placement tests. While no specific brands are endorsed, there are restrictions on the type of calculators allowed on certain classroom tests and final exams. The teacher will use the TI-84 graphing calculator family and Desmos during instruction.

Central Bucks School District – Course of Study

Unit 1: Review of Single Variable Calculus

Desired Results

Essential Questions:

- What is the difference between slope and the derivative?
- How is the Fundamental Theorem of Calculus influential in the ability to solve problems?
- How do the graphs of f, f', and f'' relate to one another?
- How is calculus used to model real-world situations?

Enduring Understandings:

Students will understand that…

- The derivative of a function expresses the instantaneous rate of change of the function.
- Rules for differentiating a function, differentiation techniques for various function types, and implicit differentiation.
- The integral of a function expresses the net change of a function over time.
- Rules for integrating a function, integration techniques for various function types, integration by parts, tabular integration, and integration by substitution.
- Using conceptual ideas of calculus to model and explore real-world situations.

Skills:

Students will be able to…

- Use a derivative as the instantaneous rate of change of a function to evaluate the slope of a function at a given value.
- Apply the rules of differentiation, including the chain rule and implicit differentiation, to derive functions.
- Use an integral as net change to determine the amount of change of a function over a specified interval.
- Construct three-dimensional shapes by revolving two-dimensional surfaces and using integration to determine the volume and surface area of the shape.
- Apply methods of integration, including substitution, integration by parts, and partial fractions, to integrate functions.

Standards:

Priority Standards

CC.2.2.HS.C.1 – Use function notation and interpret functions in context
CC.2.2.HS.C.2 – Analyze functions and multiple representations
CC.2.2.HS.C.6 – Interpret functions in context
CC.2.4.HS.B.1 – Analyze rates of change and use them to solve problems
CC.2.4.HS.B.2 – Build and refine models using functions

Supporting Standards

CC.2.1.HS.F.4 – Units and quantities
CC.2.2.HS.D.1–D.3 – Expression structure and manipulation

Acceptable Evidence

Acceptable Evidence

Performance-Based Assessments

- Multi-step problem sets requiring symbolic, numerical, and graphical solutions
- Extended-response questions requiring justification of methods and interpretation of results

- Application problems involving physical systems (motion in space, mass/density, fluid flow, etc.)
- Technology-based explorations (Desmos, graphing calculators, 3D visualization tools) analyzing multivariable functions or vector fields
- Mathematical modeling tasks requiring formulation, solution, and interpretation of real-world scenarios
- Written explanations of key theorems (e.g., Fundamental Theorem for Line Integrals, Green's Theorem) and their applications
- Projects involving visualization and analysis of surfaces, vector fields, or parametric motion

Formative Assessments

- Conceptual check-ins focusing on interpretation (not just computation)
- Error analysis of multistep calculus problems
- Peer discussion and critique of solution strategies
- Short written justifications of reasoning (e.g., existence of limits, continuity, or extrema)
- Technology-based quick tasks (graphing level curves, vector fields, parametric motion)
- Whiteboard work emphasizing explanation of methods and results
- Exit tasks requiring synthesis of concepts (e.g., connecting geometry and calculus)

Summative Assessments

- Unit assessments combining:
    - Analytical computation
    - Conceptual reasoning
    - Application/modeling tasks
- Free-response problems modeled after college-level or AP-style rigor
- Cumulative assessments incorporating multiple representations (graphical, algebraic, numerical)
- Projects or investigations (e.g., optimization with constraints, flux across a surface, motion in space)

Suggested learning experiences and instruction

Suggested Learning Experiences and Instruction

- Engage students in inquiry-based exploration of multivariable concepts (e.g., investigating level curves before formal definitions)
- Use multiple representations (graphical, algebraic, numerical, and verbal) to deepen understanding of functions of several variables
- Incorporate 3D visualization tools (Desmos 3D, graphing calculators, dynamic geometry software) to explore surfaces, vector fields, and motion in space
- Facilitate collaborative problem-solving centered on complex, multi-step applications
- Emphasize conceptual understanding alongside procedural fluency, particularly for topics such as limits in multiple variables, gradient vectors, and integrals
- Use real-world modeling tasks involving physics, engineering, and data (e.g., center of mass, circulation, optimization under constraints)
- Promote mathematical discourse, requiring students to justify reasoning, critique peers, and explain interpretations
- Integrate technology strategically to explore concepts that are difficult to visualize analytically (e.g., curl, divergence, parametric motion)
- Scaffold instruction from geometric intuition → analytic formulation → application
- Provide opportunities for productive struggle with open-ended problems and investigations
- Use case studies or applied scenarios (e.g., fluid flow, electromagnetic fields, optimization problems) to connect abstract ideas to real-world contexts
- Incorporate error analysis and debugging of solutions, especially in multistep symbolic work

Differentiation:

- Work with students in flexible small groups to provide targeted intervention, reteaching, enrichment, or extension.
- Provide additional scaffolds such as guided notes, visual representations, graphing technology, structured problem-solving frameworks, and worked examples for students who need support.
- Differentiate through extension tasks (proof-based reasoning, advanced applications) and targeted support (guided problem structures, visual scaffolds).
- Provide opportunities for students ready for additional challenge to explore more complex applications, alternative solution methods, and extended modeling tasks.

For Language Development:
- Create and maintain a student generated glossary of Calculus III terminology, symbols, definitions, and examples.
- Provide opportunities for students to explain mathematical reasoning using appropriate academic vocabulary in both written and verbal formats.
- Use graphic organizers, visual models, and structured notetaking to connect vocabulary with concepts.
- Develop study guides with clear definitions, example problems, and opportunities to practice mathematical communication.

For Enrichment:
- Meet in small groups with students who are ready for enrichment to provide instruction on advanced applications of course concepts followed by challenging extension activities.
- Allow students to pursue more rigorous problem sets that require deeper analysis, proof, justification, or multiple solution approaches.
- Provide opportunities for students to investigate real world applications of multivariable calculus in fields such as physics, engineering, economics, computer graphics, and data science.
- Encourage students to complete enrichment projects.

Unit 2: Three-Dimensional Space

Desired Results

Essential Questions:
- How can the equations of functions in three dimensions be constructed visually?
- How can we create equations for three-dimensional space?
- What are the defining characteristics for quadric surfaces and how are they determined?
- What is the right-hand rule for vectors and how does it apply in determining a solution?

Enduring Understandings:
Students will understand that…
- The geometry of three-dimensional space includes representing lines and planes in three-dimensional space.
- Constructing the equations of three-dimensional figures in space includes points, lines, and planes, as well as the equations of quadric surfaces.
- Definition of a vector and the properties of vector addition and multiplication.
- The dot product and cross product of two vectors can be used to create orthogonal projections, determine the angle between two vectors, or create the perpendicular to the intersection of the vectors.
- The cross product is the vector perpendicular to two given vectors, whose length is the area of the parallelogram determined by the vectors.

Skills:
Students will be able to…
- Determine the distance between two points in three-dimensional space.
- Use formulas for the dot product and cross product to determine missing values in a problem.
- Determine the equation for a three-dimensional object or quadric surface using defining characteristics.
- Determine the equation of a line or plane in three-dimensional space.
- Recognize that quadric surfaces are created from equations of two-dimensional conic sections.

Standards:
Priority Standards
CC.2.3.HS.A.3 – Use coordinate geometry to verify geometric relationships
CC.2.3.HS.A.5 – Apply geometric concepts in modeling situations
CC.2.3.HS.A.14 – Apply geometric concepts to real-world problems
CC.2.4.HS.B.1 – Quantitative relationships in modeling

Supporting Standards
CC.2.1.HS.F.2–F.4 – Quantitative reasoning and units
CC.2.2.HS.C.2 – Function representations

Acceptable Evidence

Acceptable Evidence

Performance-Based Assessments

- Multi-step problem sets requiring symbolic, numerical, and graphical solutions
- Extended-response questions requiring justification of methods and interpretation of results
- Application problems involving physical systems (motion in space, mass/density, fluid flow, etc.)
- Technology-based explorations (Desmos, graphing calculators, 3D visualization tools) analyzing multivariable functions or vector fields
- Mathematical modeling tasks requiring formulation, solution, and interpretation of real-world scenarios
- Written explanations of key theorems (e.g., Fundamental Theorem for Line Integrals, Green's Theorem) and their applications
- Projects involving visualization and analysis of surfaces, vector fields, or parametric motion

Formative Assessments

- Conceptual check-ins focusing on interpretation (not just computation)
- Error analysis of multistep calculus problems
- Peer discussion and critique of solution strategies
- Short written justifications of reasoning (e.g., existence of limits, continuity, or extrema)
- Technology-based quick tasks (graphing level curves, vector fields, parametric motion)
- Whiteboard work emphasizing explanation of methods and results
- Exit tasks requiring synthesis of concepts (e.g., connecting geometry and calculus)

Summative Assessments

- Unit assessments combining:
    - Analytical computation
    - Conceptual reasoning
    - Application/modeling tasks
- Free-response problems modeled after college-level or AP-style rigor
- Cumulative assessments incorporating multiple representations (graphical, algebraic, numerical)
- Projects or investigations (e.g., optimization with constraints, flux across a surface, motion in space)

Suggested learning experiences and instruction

Suggested Learning Experiences and Instruction

- Engage students in inquiry-based exploration of multivariable concepts (e.g., investigating level curves before formal definitions)
- Use multiple representations (graphical, algebraic, numerical, and verbal) to deepen understanding of functions of several variables
- Incorporate 3D visualization tools (Desmos 3D, graphing calculators, dynamic geometry software) to explore surfaces, vector fields, and motion in space
- Facilitate collaborative problem-solving centered on complex, multi-step applications
- Emphasize conceptual understanding alongside procedural fluency, particularly for topics such as limits in multiple variables, gradient vectors, and integrals
- Use real-world modeling tasks involving physics, engineering, and data (e.g., center of mass, circulation, optimization under constraints)
- Promote mathematical discourse, requiring students to justify reasoning, critique peers, and explain interpretations
- Integrate technology strategically to explore concepts that are difficult to visualize analytically (e.g., curl, divergence, parametric motion)
- Scaffold instruction from geometric intuition → analytic formulation → application
- Provide opportunities for productive struggle with open-ended problems and investigations
- Use case studies or applied scenarios (e.g., fluid flow, electromagnetic fields, optimization problems) to connect abstract ideas to real-world contexts
- Incorporate error analysis and debugging of solutions, especially in multistep symbolic work

Differentiation:

- Work with students in flexible small groups to provide targeted intervention, reteaching, enrichment, or extension.
- Provide additional scaffolds such as guided notes, visual representations, graphing technology, structured problem-solving frameworks, and worked examples for students who need support.
- Differentiate through extension tasks (proof-based reasoning, advanced applications) and targeted support (guided problem structures, visual scaffolds)
- Provide opportunities for students ready for additional challenge to explore more complex applications, alternative solution methods, and extended modeling tasks.

For Language Development:

- Create and maintain a student generated glossary of Calculus III terminology, symbols, definitions, and examples.
- Provide opportunities for students to explain mathematical reasoning using appropriate academic vocabulary in both written and verbal formats.
- Use graphic organizers, visual models, and structured notetaking to connect vocabulary with concepts.
- Develop study guides with clear definitions, example problems, and opportunities to practice mathematical communication.

For Enrichment:

- Meet in small groups with students who are ready for enrichment to provide instruction on advanced applications of course concepts followed by challenging extension activities.
- Allow students to pursue more rigorous problem sets that require deeper analysis, proof, justification, or multiple solution approaches.
- Provide opportunities for students to investigate real world applications of multivariable calculus in fields such as physics, engineering, economics, computer graphics, and data science.
- Encourage students to complete enrichment projects.

Unit 3: Vector-Valued Functions

Desired Results

Essential Questions:

- How are vector functions used to model motion through space?
- How are previously developed concepts of calculus applied to vector functions?
- How can vector functions be used to solve problems with real-world applications?

Enduring Understandings:

Students will understand that…

- The concepts and properties used to determine the derivative and integral of single-variable functions can be applied to determine the derivative of vector functions.
- Use computers (graphing calculators) to draw space curves given their functions.
- Determine the curvature and arc length of a curve.
- Use vector functions to describe the motion of an object in space.

Skills:

Students will be able to…

- Use rules for differentiation and integration from single-variable calculus on vector-valued functions.
- Find the tangent and normal vectors to smooth curves at a given point.
- Use algebraic operations, such as scalar multiplication, to manipulate vector functions.
- Extrapolate the formula for arc length from single-variable calculus to vector functions.
- Use the concepts of tangent and normal vectors to determine the curvature of a smooth curve.

Standards:

<u>Priority Standards</u>

CC.2.4.HS.B.1 – Interpret rates of change
CC.2.4.HS.B.2 – Build and use mathematical models
CC.2.2.HS.C.2 – Analyze functions across representations

<u>Supporting Standards</u>

CC.2.1.HS.F.4–F.5 – Units, precision
CC.2.2.HS.C.6 – Interpret functions in context

Acceptable Evidence

Acceptable Evidence

Performance-Based Assessments

- Multi-step problem sets requiring symbolic, numerical, and graphical solutions
- Extended-response questions requiring justification of methods and interpretation of results
- Application problems involving physical systems (motion in space, mass/density, fluid flow, etc.)
- Technology-based explorations (Desmos, graphing calculators, 3D visualization tools) analyzing multivariable functions or vector fields

- Mathematical modeling tasks requiring formulation, solution, and interpretation of real-world scenarios
- Written explanations of key theorems (e.g., Fundamental Theorem for Line Integrals, Green's Theorem) and their applications
- Projects involving visualization and analysis of surfaces, vector fields, or parametric motion

Formative Assessments

- Conceptual check-ins focusing on interpretation (not just computation)
- Error analysis of multistep calculus problems
- Peer discussion and critique of solution strategies
- Short written justifications of reasoning (e.g., existence of limits, continuity, or extrema)
- Technology-based quick tasks (graphing level curves, vector fields, parametric motion)
- Whiteboard work emphasizing explanation of methods and results
- Exit tasks requiring synthesis of concepts (e.g., connecting geometry and calculus)

Summative Assessments

- Unit assessments combining:
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- Free-response problems modeled after college-level or AP-style rigor
- Cumulative assessments incorporating multiple representations (graphical, algebraic, numerical)
- Projects or investigations (e.g., optimization with constraints, flux across a surface, motion in space)

Suggested learning experiences and instruction

Suggested Learning Experiences and Instruction

- Engage students in inquiry-based exploration of multivariable concepts (e.g., investigating level curves before formal definitions)
- Use multiple representations (graphical, algebraic, numerical, and verbal) to deepen understanding of functions of several variables
- Incorporate 3D visualization tools (Desmos 3D, graphing calculators, dynamic geometry software) to explore surfaces, vector fields, and motion in space
- Facilitate collaborative problem-solving centered on complex, multi-step applications
- Emphasize conceptual understanding alongside procedural fluency, particularly for topics such as limits in multiple variables, gradient vectors, and integrals
- Use real-world modeling tasks involving physics, engineering, and data (e.g., center of mass, circulation, optimization under constraints)
- Promote mathematical discourse, requiring students to justify reasoning, critique peers, and explain interpretations
- Integrate technology strategically to explore concepts that are difficult to visualize analytically (e.g., curl, divergence, parametric motion)
- Scaffold instruction from geometric intuition → analytic formulation → application
- Provide opportunities for productive struggle with open-ended problems and investigations
- Use case studies or applied scenarios (e.g., fluid flow, electromagnetic fields, optimization problems) to connect abstract ideas to real-world contexts

Differentiation:

- Work with students in flexible small groups to provide targeted intervention, reteaching, enrichment, or extension.
- Provide additional scaffolds such as guided notes, visual representations, graphing technology, structured problem-solving frameworks, and worked examples for students who need support.

- Differentiate through extension tasks (proof-based reasoning, advanced applications) and targeted support (guided problem structures, visual scaffolds).
- Provide opportunities for students ready for additional challenge to explore more complex applications, alternative solution methods, and extended modeling tasks.

For Language Development:

- Create and maintain a student generated glossary of Calculus III terminology, symbols, definitions, and examples.
- Provide opportunities for students to explain mathematical reasoning using appropriate academic vocabulary in both written and verbal formats.
- Use graphic organizers, visual models, and structured notetaking to connect vocabulary with concepts.
- Develop study guides with clear definitions, example problems, and opportunities to practice mathematical communication.

For Enrichment:

- Meet in small groups with students who are ready for enrichment to provide instruction on advanced applications of course concepts followed by challenging extension activities.
- Allow students to pursue more rigorous problem sets that require deeper analysis, proof, justification, or multiple solution approaches.
- Provide opportunities for students to investigate real world applications of multivariable calculus in fields such as physics, engineering, economics, computer graphics, and data science.
- Encourage students to complete enrichment projects.

Unit 4: Partial Derivatives

Desired Results

Essential Questions:

- What is a function of several variables?
- How do the concepts and rules of differentiation for functions of one variable extend to functions of several variables?
- How do you determine maximum, minimum, and saddle points for functions of several variables?
- What is the gradient of a function?
- How does the method of Lagrange Multipliers work?
- How can the limit of a multivariable function be shown to exist or not exist?

Enduring Understandings:

Students will understand that…

- Use level curves to visualize functions of several variables.
- Extend the ideas of implicit differentiation from single variable calculus to determine particular and partial derivatives for functions of several variables.
- Find the limit of a multivariable function.
- Use previous rules for differentiation, including the Chain Rule, to determine partial derivatives for functions of several variables.
- Use fundamental concepts of limits and continuity to determine the limit and continuity for functions of several variables.
- Determine tangent lines, planes, and linear approximations of functions of several variables.
- Use partial derivatives to create directional derivatives and the gradient vector of a function.
- Use partial derivatives to determine classifying characteristics of three-dimensional surfaces, including maximums, minimums, and saddle points.
- Utilize Lagrange multipliers in addition to partial derivatives to determine classifying characteristics of three-dimensional surfaces.

Skills:

Students will be able to…

- Sketch several levels of the curves of a function to create a visualization.
- Use a contour map to develop a rough sketch of a function of several variables.
- Use techniques of differentiation to determine tangent planes and normal lines of a given surface at a specified point or parallel to a specified direction.

Standards:

Priority Standards

CC.2.4.HS.B.1 – Analyze rates of change

CC.2.4.HS.B.2 – Build models using functions

CC.2.2.HS.C.2 – Interpret multiple representations of functions

Supporting Standards
CC.2.1.HS.F.4 – Units and interpretation
CC.2.2.HS.C.6 – Contextual interpretation

Acceptable Evidence

Acceptable Evidence

Performance-Based Assessments

- Multi-step problem sets requiring symbolic, numerical, and graphical solutions
- Extended-response questions requiring justification of methods and interpretation of results
- Application problems involving physical systems (motion in space, mass/density, fluid flow, etc.)
- Technology-based explorations (Desmos, graphing calculators, 3D visualization tools) analyzing multivariable functions or vector fields
- Mathematical modeling tasks requiring formulation, solution, and interpretation of real-world scenarios
- Written explanations of key theorems (e.g., Fundamental Theorem for Line Integrals, Green's Theorem) and their applications
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Formative Assessments

- Conceptual check-ins focusing on interpretation (not just computation)
- Error analysis of multistep calculus problems
- Peer discussion and critique of solution strategies
- Short written justifications of reasoning (e.g., existence of limits, continuity, or extrema)
- Technology-based quick tasks (graphing level curves, vector fields, parametric motion)
- Whiteboard work emphasizing explanation of methods and results
- Exit tasks requiring synthesis of concepts (e.g., connecting geometry and calculus)

Summative Assessments

- Unit assessments combining:
  - Analytical computation
  - Conceptual reasoning
  - Application/modeling tasks
- Free-response problems modeled after college-level or AP-style rigor
- Cumulative assessments incorporating multiple representations (graphical, algebraic, numerical)
- Projects or investigations (e.g., optimization with constraints, flux across a surface, motion in space)

Suggested learning experiences and instruction

Suggested Learning Experiences and Instruction

- Engage students in inquiry-based exploration of multivariable concepts (e.g., investigating level curves before formal definitions)
- Use multiple representations (graphical, algebraic, numerical, and verbal) to deepen understanding of functions of several variables
- Incorporate 3D visualization tools (Desmos 3D, graphing calculators, dynamic geometry software) to explore surfaces, vector fields, and motion in space
- Facilitate collaborative problem-solving centered on complex, multi-step applications
- Emphasize conceptual understanding alongside procedural fluency, particularly for topics such as limits in multiple variables, gradient vectors, and integrals
- Use real-world modeling tasks involving physics, engineering, and data (e.g., center of mass, circulation, optimization under constraints)
- Promote mathematical discourse, requiring students to justify reasoning, critique peers, and explain interpretations

- Integrate technology strategically to explore concepts that are difficult to visualize analytically (e.g., curl, divergence, parametric motion)
- Scaffold instruction from geometric intuition → analytic formulation → application
- Provide opportunities for productive struggle with open-ended problems and investigations
- Use case studies or applied scenarios (e.g., fluid flow, electromagnetic fields, optimization problems) to connect abstract ideas to real-world contexts
- Incorporate error analysis and debugging of solutions, especially in multistep symbolic work

Differentiation:
- Work with students in flexible small groups to provide targeted intervention, reteaching, enrichment, or extension.
- Provide additional scaffolds such as guided notes, visual representations, graphing technology, structured problem-solving frameworks, and worked examples for students who need support.
- Differentiate through extension tasks (proof-based reasoning, advanced applications) and targeted support (guided problem structures, visual scaffolds)
- Provide opportunities for students ready for additional challenge to explore more complex applications, alternative solution methods, and extended modeling tasks.

For Language Development:
- Create and maintain a student generated glossary of Calculus III terminology, symbols, definitions, and examples.
- Provide opportunities for students to explain mathematical reasoning using appropriate academic vocabulary in both written and verbal formats.
- Use graphic organizers, visual models, and structured notetaking to connect vocabulary with concepts.
- Develop study guides with clear definitions, example problems, and opportunities to practice mathematical communication.

For Enrichment:
- Meet in small groups with students who are ready for enrichment to provide instruction on advanced applications of course concepts followed by challenging extension activities.
- Allow students to pursue more rigorous problem sets that require deeper analysis, proof, justification, or multiple solution approaches.
- Provide opportunities for students to investigate real world applications of multivariable calculus in fields such as physics, engineering, economics, computer graphics, and data science.
- Encourage students to complete enrichment projects.

Unit 5: Multiple Integrals

Desired Results

Essential Questions:

- What is the definition of a multiple integral?
- How can an integral from Cartesian coordinates be converted to polar, cylindrical, or spherical coordinates?
- What are the differences between type I and type II regions?
- What are the differences among Type I, Type II, and Type III solid regions?
- How are multiple and iterated integrals applied to real-world problems?

Enduring Understandings:

Students will understand that…

- Extend the ideas of integration from single variable calculus to determine multiple integrals for functions of several variables.
- Use double integrals to determine the volume of a three-dimensional shape.
- Double integrals can be used to determine density, mass, moments, centers of mass, and probabilistic equations such as the joint density function.
- Use iterated integrals to determine the surface area of a quadric surface.
- The concept of multiple integrals can be extended from Cartesian coordinates to polar, cylindrical, and spherical coordinates.
- The substitution method from single-variable calculus is extrapolated to the change-of-variables method in multiple integrals to simplify integration.

Skills:

Students will be able to…

- When advantageous, use the ability to convert among rectangular, polar, cylindrical, and spherical coordinates to evaluate an iterated integral effectively and efficiently.
- Use iterated integrals to determine quantities, mass, density, and related measures of lamina in three-dimensional space.
- Describe the region given by a definite iterated integral and describe the volume of the solid generated by evaluating the integral.
- Use change of variables and appropriate transformations to evaluate multiple integrals.

Standards:

<u>Priority Standards</u>

CC.2.3.HS.A.12 – Explain and use volume formulas
CC.2.3.HS.A.13 – Analyze 2D–3D relationships
CC.2.4.HS.B.1 – Quantitative reasoning and accumulation
CC.2.4.HS.B.2 – Model real-world phenomena

Supporting Standards
CC.2.1.HS.F.4 – Units
CC.2.2.HS.C.2 – Function representations

Acceptable Evidence

Acceptable Evidence

Performance-Based Assessments

- Multi-step problem sets requiring symbolic, numerical, and graphical solutions
- Extended-response questions requiring justification of methods and interpretation of results
- Application problems involving physical systems (motion in space, mass/density, fluid flow, etc.)
- Technology-based explorations (Desmos, graphing calculators, 3D visualization tools) analyzing multivariable functions or vector fields
- Mathematical modeling tasks requiring formulation, solution, and interpretation of real-world scenarios
- Written explanations of key theorems (e.g., Fundamental Theorem for Line Integrals, Green's Theorem) and their applications
- Projects involving visualization and analysis of surfaces, vector fields, or parametric motion

Formative Assessments

- Conceptual check-ins focusing on interpretation (not just computation)
- Error analysis of multistep calculus problems
- Peer discussion and critique of solution strategies
- Short written justifications of reasoning (e.g., existence of limits, continuity, or extrema)
- Technology-based quick tasks (graphing level curves, vector fields, parametric motion)
- Whiteboard work emphasizing explanation of methods and results
- Exit tasks requiring synthesis of concepts (e.g., connecting geometry and calculus)

Summative Assessments

- Unit assessments combining:
  - Analytical computation
  - Conceptual reasoning
  - Application/modeling tasks
- Free-response problems modeled after college-level or AP-style rigor
- Cumulative assessments incorporating multiple representations (graphical, algebraic, numerical)
- Projects or investigations (e.g., optimization with constraints, flux across a surface, motion in space)

Suggested learning experiences and instruction

Suggested Learning Experiences and Instruction

- Engage students in inquiry-based exploration of multivariable concepts (e.g., investigating level curves before formal definitions)
- Use multiple representations (graphical, algebraic, numerical, and verbal) to deepen understanding of functions of several variables
- Incorporate 3D visualization tools (Desmos 3D, graphing calculators, dynamic geometry software) to explore surfaces, vector fields, and motion in space
- Facilitate collaborative problem-solving centered on complex, multi-step applications
- Emphasize conceptual understanding alongside procedural fluency, particularly for topics such as limits in multiple variables, gradient vectors, and integrals
- Use real-world modeling tasks involving physics, engineering, and data (e.g., center of mass, circulation, optimization under constraints)
- Promote mathematical discourse, requiring students to justify reasoning, critique peers, and explain interpretations

- Integrate technology strategically to explore concepts that are difficult to visualize analytically (e.g., curl, divergence, parametric motion)
- Scaffold instruction from geometric intuition → analytic formulation → application
- Provide opportunities for productive struggle with open-ended problems and investigations
- Use case studies or applied scenarios (e.g., fluid flow, electromagnetic fields, optimization problems) to connect abstract ideas to real-world contexts
- Incorporate error analysis and debugging of solutions, especially in multistep symbolic work

Differentiation:
- Work with students in flexible small groups to provide targeted intervention, reteaching, enrichment, or extension.
- Provide additional scaffolds such as guided notes, visual representations, graphing technology, structured problem-solving frameworks, and worked examples for students who need support.
- Differentiate through extension tasks (proof-based reasoning, advanced applications) and targeted support (guided problem structures, visual scaffolds)
- Provide opportunities for students ready for additional challenge to explore more complex applications, alternative solution methods, and extended modeling tasks.

For Language Development:
- Create and maintain a student generated glossary of Calculus III terminology, symbols, definitions, and examples.
- Provide opportunities for students to explain mathematical reasoning using appropriate academic vocabulary in both written and verbal formats.
- Use graphic organizers, visual models, and structured notetaking to connect vocabulary with concepts.
- Develop study guides with clear definitions, example problems, and opportunities to practice mathematical communication.

For Enrichment:
- Meet in small groups with students who are ready for enrichment to provide instruction on advanced applications of course concepts followed by challenging extension activities.
- Allow students to pursue more rigorous problem sets that require deeper analysis, proof, justification, or multiple solution approaches.
- Provide opportunities for students to investigate real world applications of multivariable calculus in fields such as physics, engineering, economics, computer graphics, and data science.
- Encourage students to complete enrichment projects.

Unit 6: Topics in Vector Calculus

Desired Results

Essential Questions:

- What is a vector field?
- What is the fundamental theorem for line integrals?
- What is Green's Theorem?
- What is the relationship between curl and divergence?
- What is the relationship among the Fundamental Theorem of Calculus, Fundamental Theorem for Line Integrals, Green's Theorem, Stokes' Theorem, and the Divergence Theorem?

Enduring Understandings:

Students will understand that…

- Integrating a function over a curve C in place of a set interval [a, b] produces the line integral of the function.
  - The fundamental theorem for line integrals relates a function's gradient and the integral of the function along a smooth curve. This creates the relationship that the line integral of the gradient of a function is the net change in the function.
- Green's Theorem provides the relationship between a line integral around a simple closed curve and a double integral over the plane bounded by the curve.
- The curl of a function is the vector field defined by the cross product of the function and its gradient.
- The divergence of a function is the scalar field defined by the dot product of the function and its gradient.

Skills:

Students will be able to…

- Evaluate the line integral of a smooth, simple, closed plane curve of a piecewise function under basic operations.
- Evaluate the line integral of a function as applied to various geometric situations and oriented surfaces.
- Given a vector field, curve, and point, determine whether the line integral of the function is positive, negative, or zero.
- Show that **F** is a conservative vector field and determine the function. Use a conservative vector field to evaluate a line integral.
- Use Green's Theorem, Stokes' Theorem, and the Divergence Theorem as applied to various geometric shapes and oriented surfaces.

Standards:

<u>Priority Standards</u>

CC.2.4.HS.B.1 – Quantitative relationships and change
CC.2.4.HS.B.2 – Mathematical modeling
CC.2.3.HS.A.5 – Geometric modeling

Supporting Standards
CC.2.1.HS.F.4–F.5 – Units and precision
CC.2.2.HS.C.2 – Interpret functions

Acceptable Evidence

Acceptable Evidence

Performance-Based Assessments

- Multi-step problem sets requiring symbolic, numerical, and graphical solutions
- Extended-response questions requiring justification of methods and interpretation of results
- Application problems involving physical systems (motion in space, mass/density, fluid flow, etc.)
- Technology-based explorations (Desmos, graphing calculators, 3D visualization tools) analyzing multivariable functions or vector fields
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Formative Assessments

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Summative Assessments

- Unit assessments combining:
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- Free-response problems modeled after college-level or AP-style rigor
- Cumulative assessments incorporating multiple representations (graphical, algebraic, numerical)
- Projects or investigations (e.g., optimization with constraints, flux across a surface, motion in space)

Suggested learning experiences and instruction

Suggested Learning Experiences and Instruction

- Engage students in inquiry-based exploration of multivariable concepts (e.g., investigating level curves before formal definitions)
- Use multiple representations (graphical, algebraic, numerical, and verbal) to deepen understanding of functions of several variables
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- Integrate technology strategically to explore concepts that are difficult to visualize analytically (e.g., curl, divergence, parametric motion)
- Scaffold instruction from geometric intuition → analytic formulation → application
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- Use case studies or applied scenarios (e.g., fluid flow, electromagnetic fields, optimization problems) to connect abstract ideas to real-world contexts
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Differentiation:
- Work with students in flexible small groups to provide targeted intervention, reteaching, enrichment, or extension.
- Provide additional scaffolds such as guided notes, visual representations, graphing technology, structured problem-solving frameworks, and worked examples for students who need support.
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For Language Development:
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Unit 7: Second Order Differential Equations

Desired Results

Essential Questions:

- What is the general form of the auxiliary equation for a second-order differential equation?
- How do the roots of the auxiliary equations affect the form of the solution to a second-order differential equation?
- How is an initial value problem solved?
- How are power series used to solve a differential equation?

Enduring Understandings:

Students will understand that…

- A second order linear differential equation is of the form $P(x)\left(\frac{d^2y}{dx^2}\right) + Q(x)\left(\frac{dy}{dx}\right) + R(x)y = G(x)$, where P, Q, R and G are continuous functions.
- Second-order differential equations are either homogeneous, in the case , or nonhomogeneous, .
- The methods of undetermined coefficients and variation of parameters are used to solve nonhomogeneous linear equations.
- Second-order differential equations are used to model various real-world situations, such as vibrations of strings and electric circuits.
- The solution to a second order differential equation can be represented as an infinite series.

Skills:

Students will be able to…

- Solve a second-order differential equation using an auxiliary equation, characteristic equation, or method of variation of parameters for general and specific solutions.
- Apply second-order differential equations to real-world initial value problems, such as vibration and circuits, to solve for initial and approximate values.
- Use power series to approximate variable coefficients in the general-form solution of both homogeneous and nonhomogeneous second-order differential equations.

Standards:

Priority Standards

CC.2.4.HS.B.1 – Analyze rates of change
CC.2.4.HS.B.2 – Use functions to model systems
CC.2.2.HS.C.6 – Interpret functions in context

Supporting Standards

CC.2.1.HS.F.4 – Units and modeling
CC.2.2.HS.C.2 – Analyze functions

Acceptable Evidence

Acceptable Evidence

Performance-Based Assessments

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Suggested learning experiences and instruction

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Appendix – Table of Contents for Teacher Resources

Intranet Resources:

- Course of Study

Internet Resources:

- Calculus using the TI-84: http://education.ti.com/html/t3_free_courses/calculus84_online/index.html
- MIT Open Courseware: http://ocw.mit.edu/courses/mathematics/18-01-single-variable-calculus-fall-2006/
- Wolfram Demonstrations Project: http://demonstrations.wolfram.com/
- Wolfram Alpha – computational search engine: www.wolframalpha.com
- Mathworld: www.mathworld.wolfram.com
- Wolfram Online Integrator: http://integrals.wolfram.com/index.jsp?expr=Ln%5Bx%5D&random=false